Low-Overhead Magic State Distillation with Color Codes

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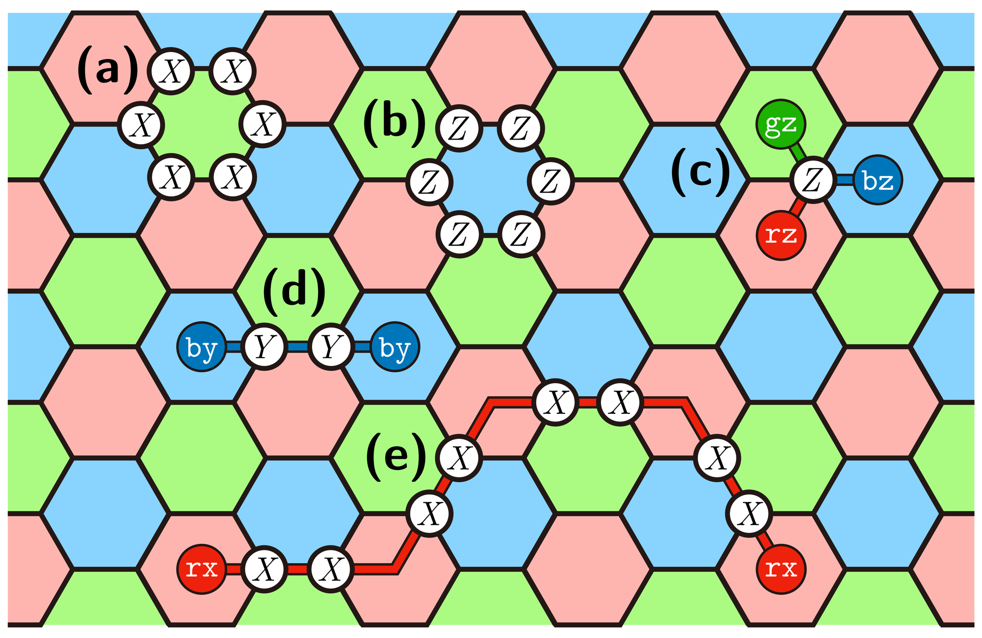
Summary

- **2D color codes** [1] offer advantages such as transversal Clifford gates, higher encoding rates than surface codes, and efficient lattice surgery with lower spacetime overhead [2], despite a lower error threshold compared to surface codes.
- **Motivation**: Magic state distillation (MSD) is essential for universal quantum computing but highly costly due to its demand for many logical operations. We should tailor & optimise it to a specific code we want to use. Such optimisation has been studied well for surface codes by Litinski [5, 6]. How about for color codes?
- We propose **two MSD schemes tailored to 2D color codes**:
  - **Single-level scheme**: A scheme based on faulty T-measurement, optimised for higher target error rates ($\gtrsim 35p^3$ for physical error rate p).
  - **Combined scheme**: A hybrid scheme combining Scheme 1 with distillation-free magic state preparation [4], achieving significantly low error rates (e.g., $\sim 10^{-19}$ for $p = 10^{-4}$).
- Both schemes achieve lower spacetime costs compared to existing MSD methods for color codes [3], reducing resources by up to two orders of magnitude for a given target error rate.

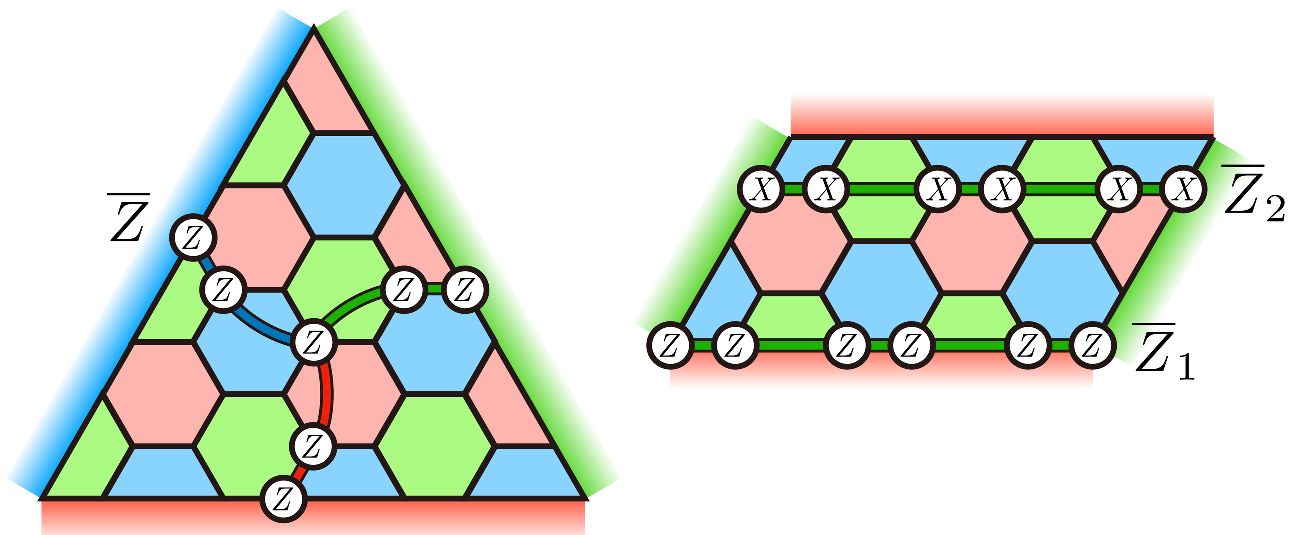
2D color codes



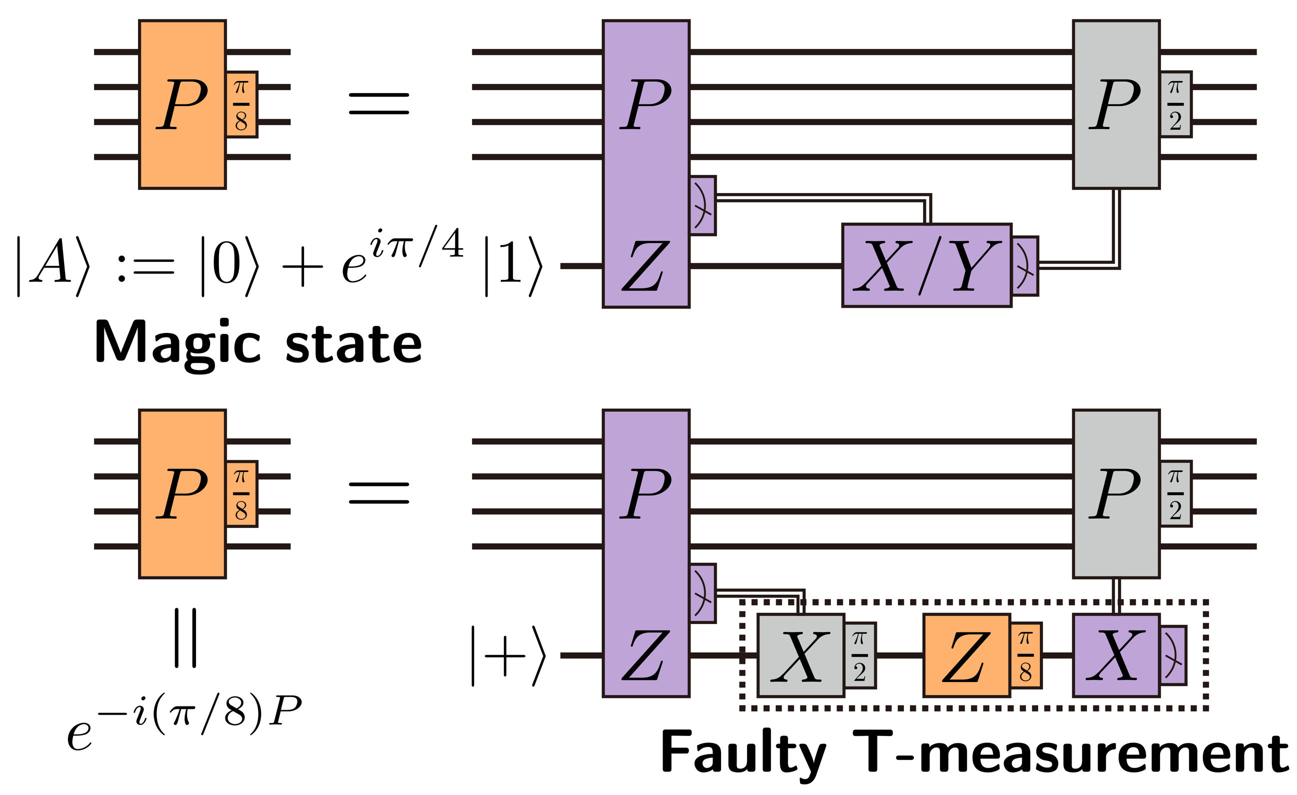
- **2D color code lattice**
  - **3-valent**: Each vertex is connected with three edges.
  - **3-colourable**: Each face can be coloured with one of three colours ($\texttt{r}$, $\texttt{g}$, $\texttt{b}$) in a way that adjacent faces have different colours.
- **Stabiliser generators**

$$\text{For each face } f, \quad S_f^X := \prod_{v\in f} X_v, \quad S_f^Z := \prod_{v\in f} Z_v$$
$$\rightarrow S_f^X |\psi\rangle = |\psi\rangle, \ S_f^Z |\psi\rangle = |\psi\rangle.$$

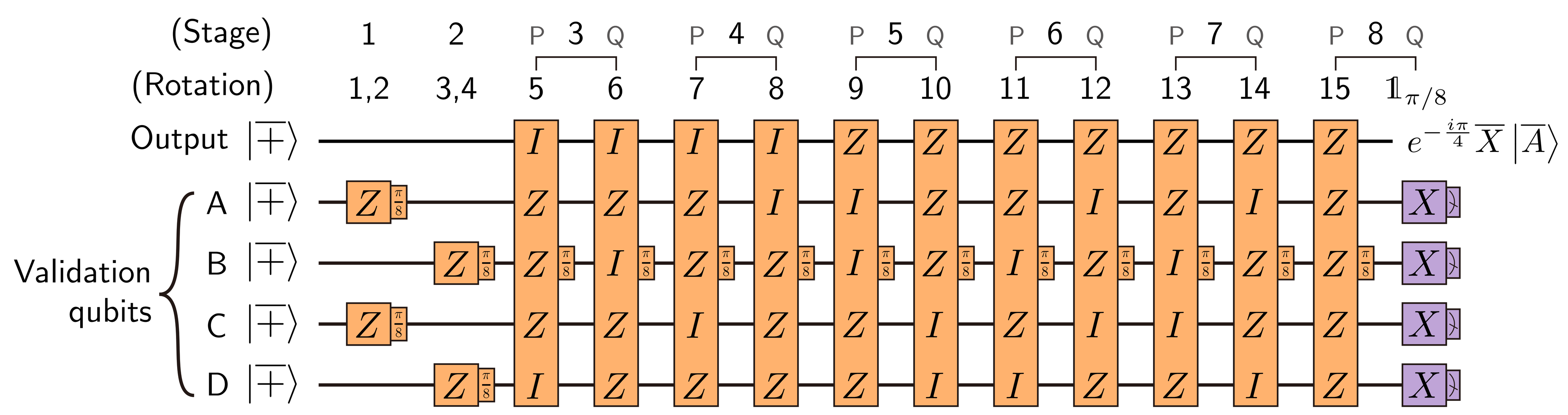
- **Logical qubit**

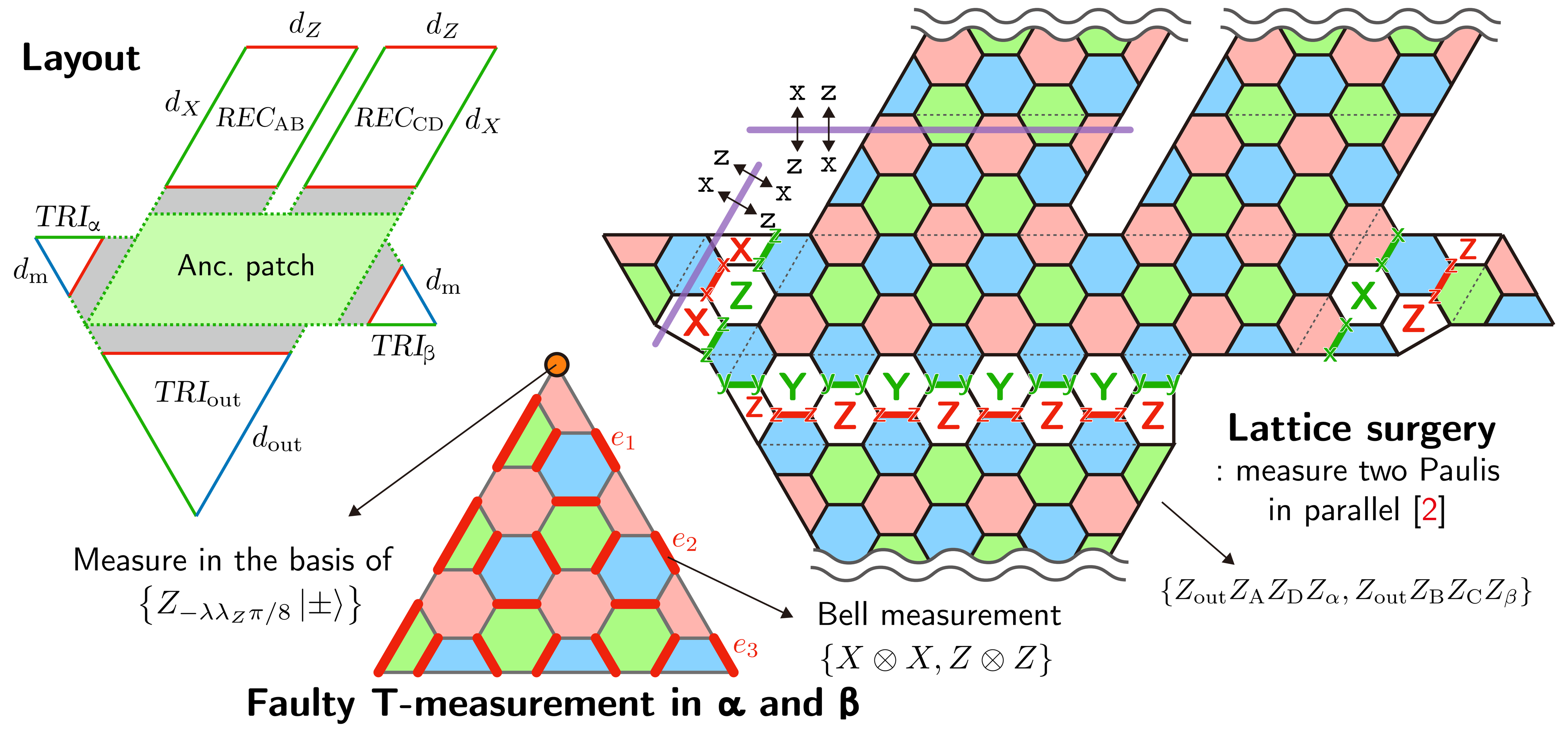


Implementing non-Clifford gates

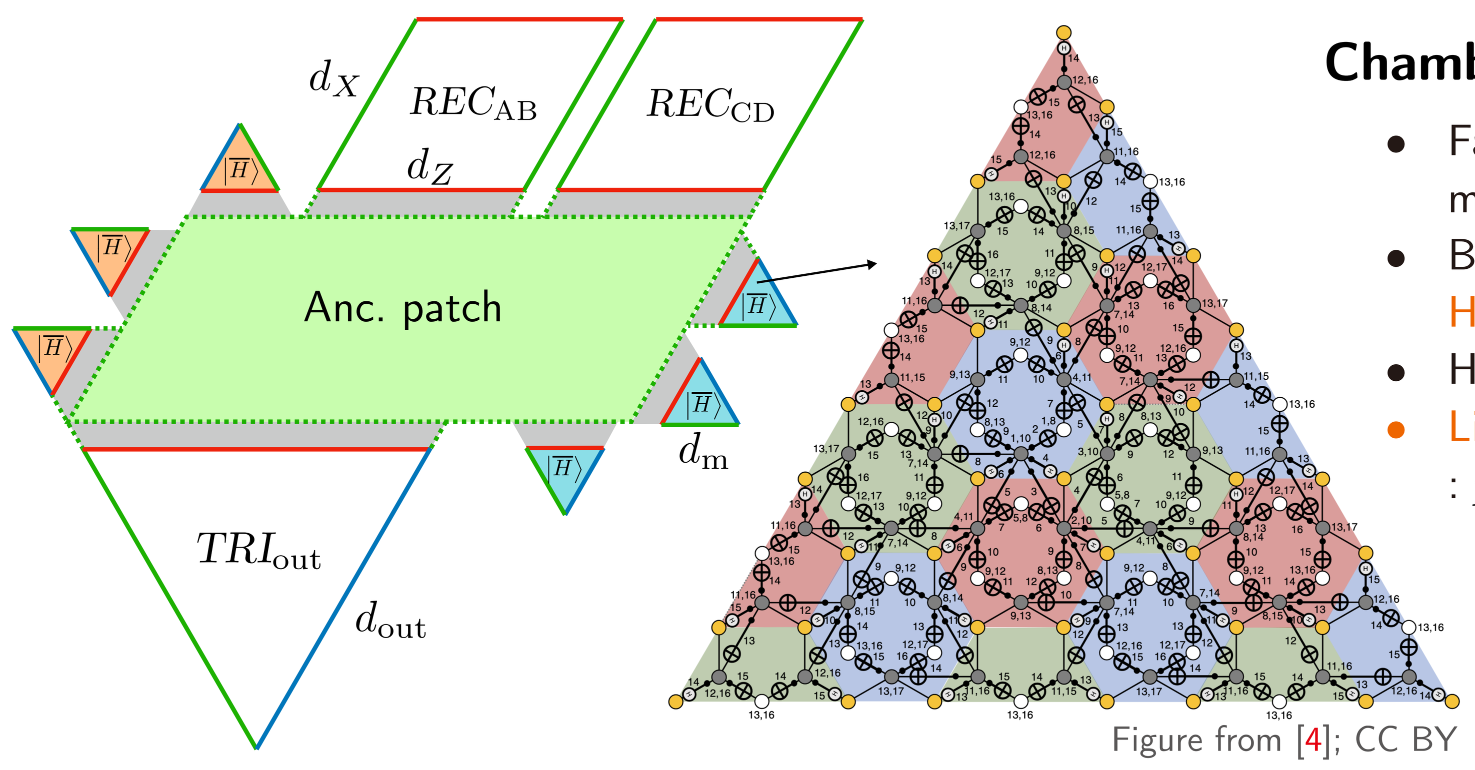


Single-level scheme: MSD based on faulty T-measurement





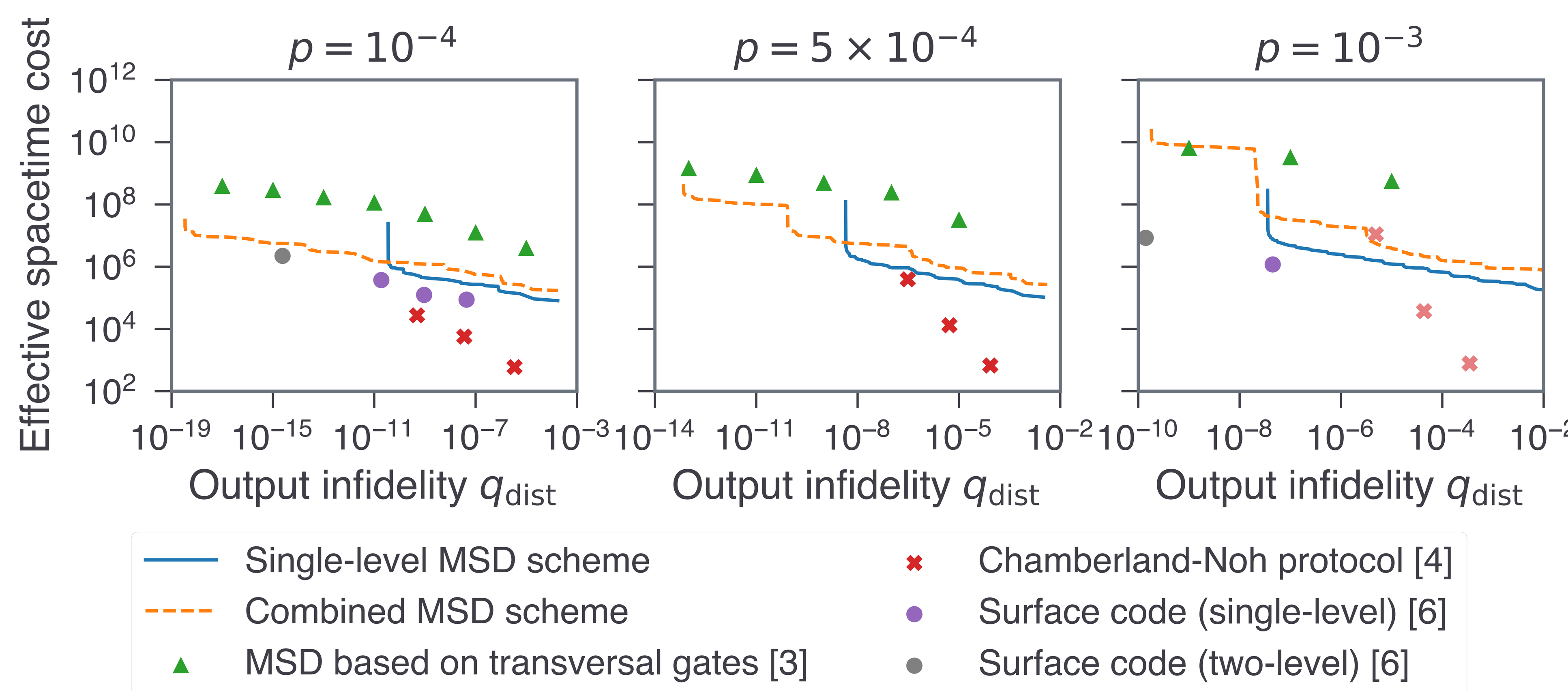
Combined scheme: MSD combined with dist.-free magic state preparation



Chamberland-Noh protocol [4]

- Fault-tolerantly prepare magic states without MSD.
- Based on transversal logical Hadamard measurement.
- Highly resource-efficient.
- Limited output quality: $p_L \gtrsim 10^{-10}$ when $p = 10^{-4}$

Numerical analysis



References

[1] H. Bombin & M. A. Martin-Delgado, Phys. Rev. Lett. **97**, 180501 (2006).

[2] F. Thomsen *et al.*, Phys. Rev. Research **6**, 043125 (2024).

[3] M. E. Beverland *et al.*, PRX Quantum **2**, 020341 (2021).

[4] C. Chamberland & K. Noh, npj Quantum Inf. **6**, 91 (2020).

[5] D. Litinski, Quantum 3, **128** (2019).

[6] D. Litinski, Quantum 3, **205** (2019).

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