GRAPH-THEORETICAL OPTIMIZATION OF FUSION-BASED GRAPH STATE GENERATION

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SUMMARY

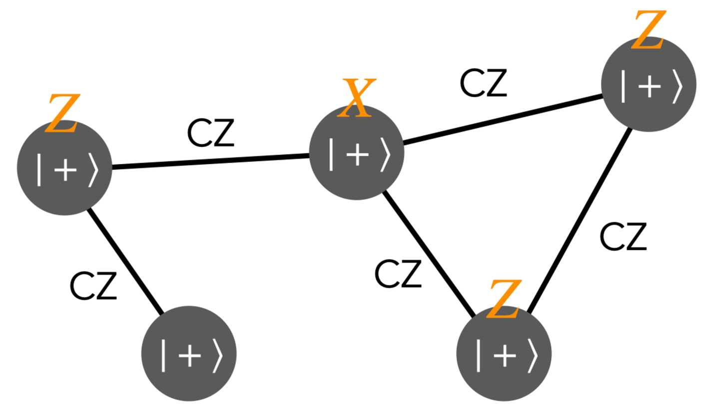
- **Graph states**: Resources for various quantum information task including measurement-based quantum computing, fusion-based quantum computing, quantum repeaters, and quantum metrology
- In linear optical systems, graph states can be generated by **type-II fusion operations** → **Non-deterministic**, thus large graph states are difficult to generate.
- We propose **a graph-theoretical strategy to effectively optimize the generation of any graph state via type-II fusions.**
- Main idea: find a **graph state equivalent to the desired graph state** under local Clifford gates and type-II fusions but easier to generate → **Unraveling**
- We expect that our strategy and software will assist researchers in developing and accessing experimentally viable schemes that use photonic graph states.

BACKGROUND

- **Graph state:** For a graph $G=(V,E)$,

$$|G\rangle_V := \prod_{v_1,v_2\in E} U_{CZ}(v_1,v_2) \bigotimes_{v\in V} |+\rangle_v$$

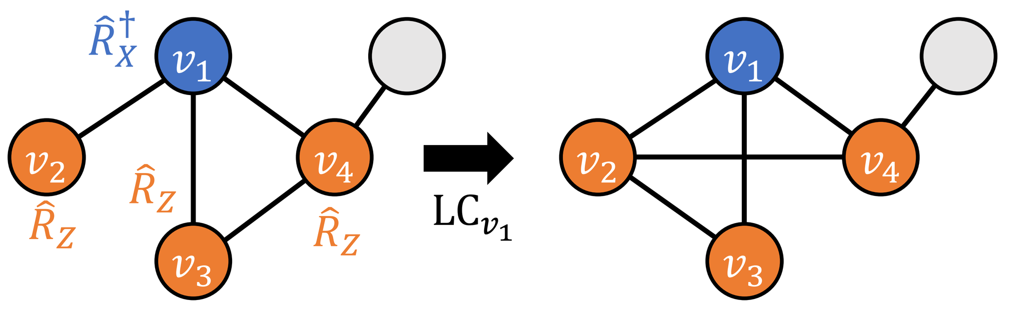
$$\forall v\in V,\ S_v|G\rangle := \left(X_v \prod_{u\in \mathrm{adj}(v)} Z_u\right)|G\rangle = |G\rangle$$



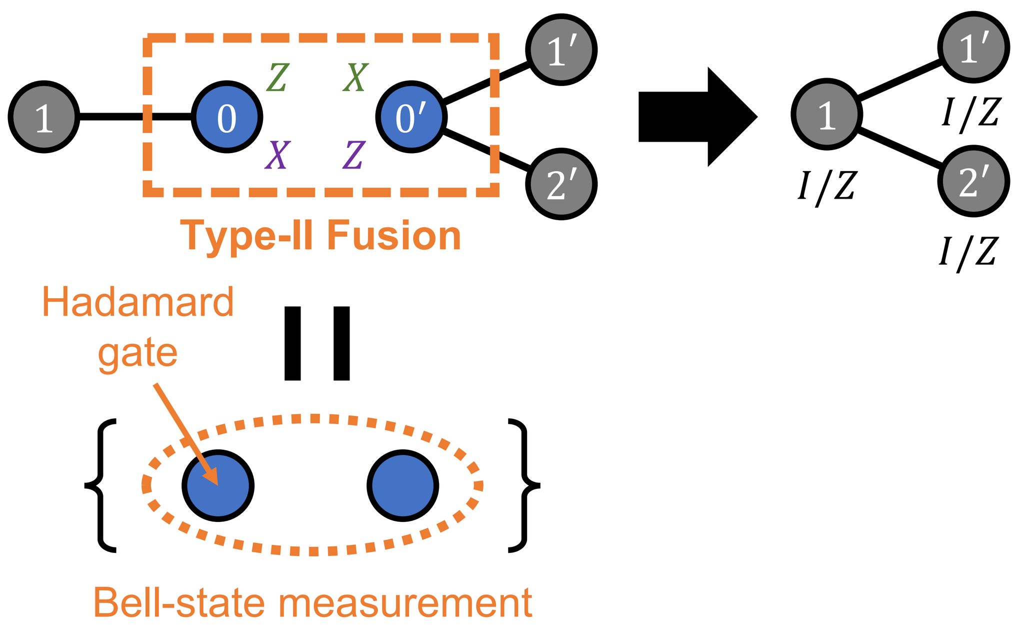
→ Used in measurement-based quantum computing [1,2], fusion-based quantum computing [3], quantum repeaters [4], quantum metrology [5], etc.

- Equivalence of graph states under local Clifford gates:

$$\exp\left[-i\frac{\pi}{4}X_v\right]\prod_{u\in \mathrm{adj}(v)}\exp\left[i\frac{\pi}{4}Z_u\right]|G\rangle = |\mathrm{LC}_v(G)\rangle$$ → **Local complementation** (LC)

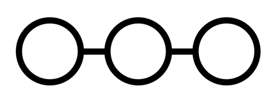


- **Type-II fusion** [6]: Measuring $\{X\otimes Z, Z\otimes X\}$
  → Connect/disconnect every pair of adjacent vertices of two vertices



$$p_{\mathrm{succ}} = \frac{(1-\eta)^2}{2}$$

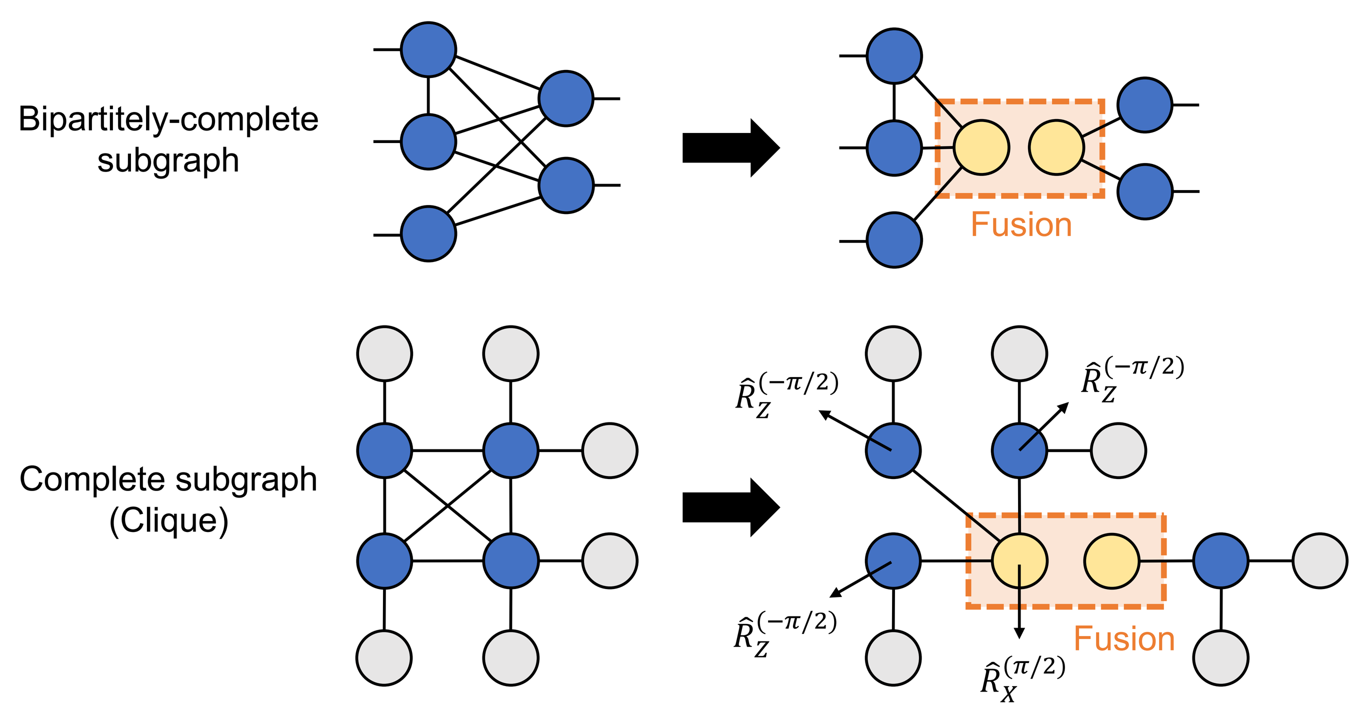
OVERVIEW OF THE STRATEGY



- Basic resource state: **three-qubit linear graph state** $|G_*^{(3)}\rangle := |{+}0{+}\rangle + |{-}1{-}\rangle$
  (Can be generated with a success rate of $1/32$ linear-optically [7])
- **Resource cost** Q: Average number of $|G_*^{(3)}\rangle$'s required to successfully generate one $|G\rangle$ state through post-selection

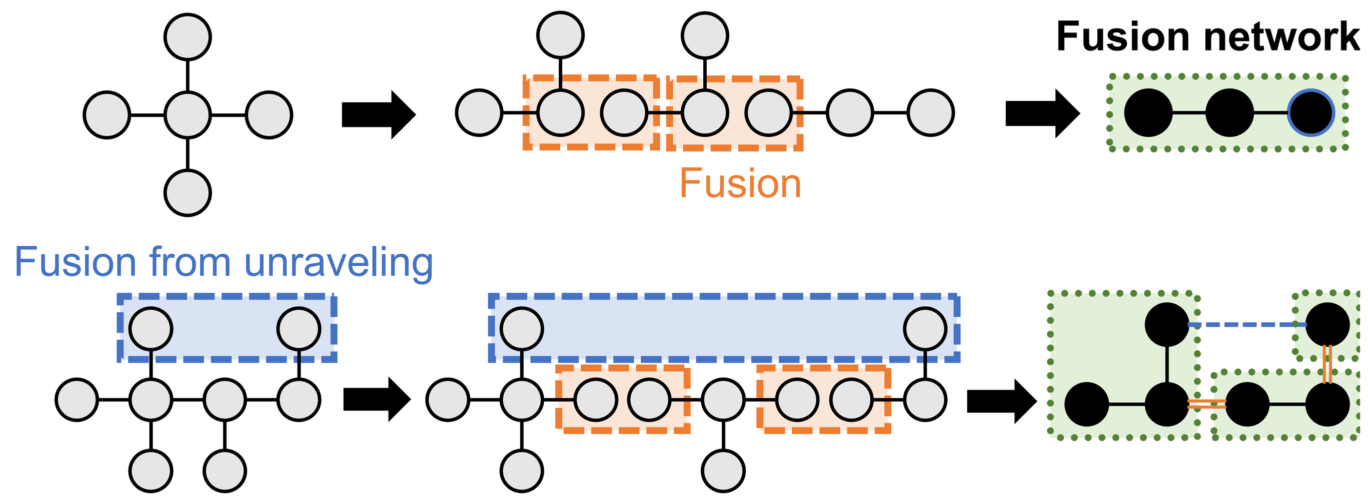
1. Simplify the graph of the desired graph state via **unraveling**.
2. Construct a **fusion network**.
3. Determine the **fusion order** and calculate the **resource cost** Q.
4. Iterate 1–3 a sufficient number of times and select the best one.

STAGE 1: UNRAVELING

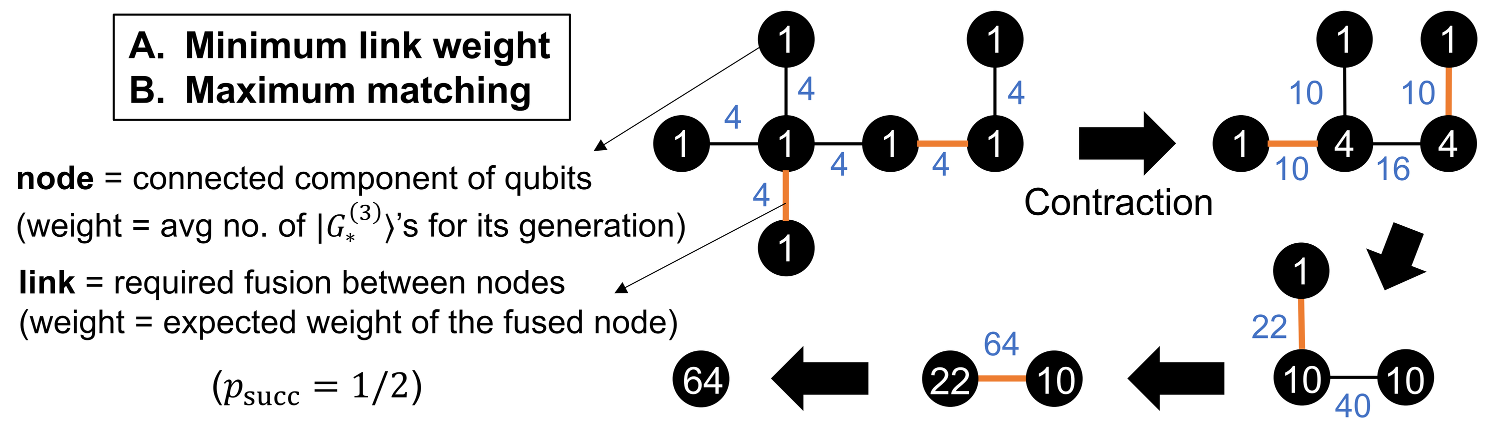


- Finding all non-overlapping bipartitely-complete subgraphs → $O\left(|V|\,d_{\max}^4\right)$
- Finding all non-overlapping cliques [8] → $O(\mathrm{poly}(N_{\mathrm{clique}}))$

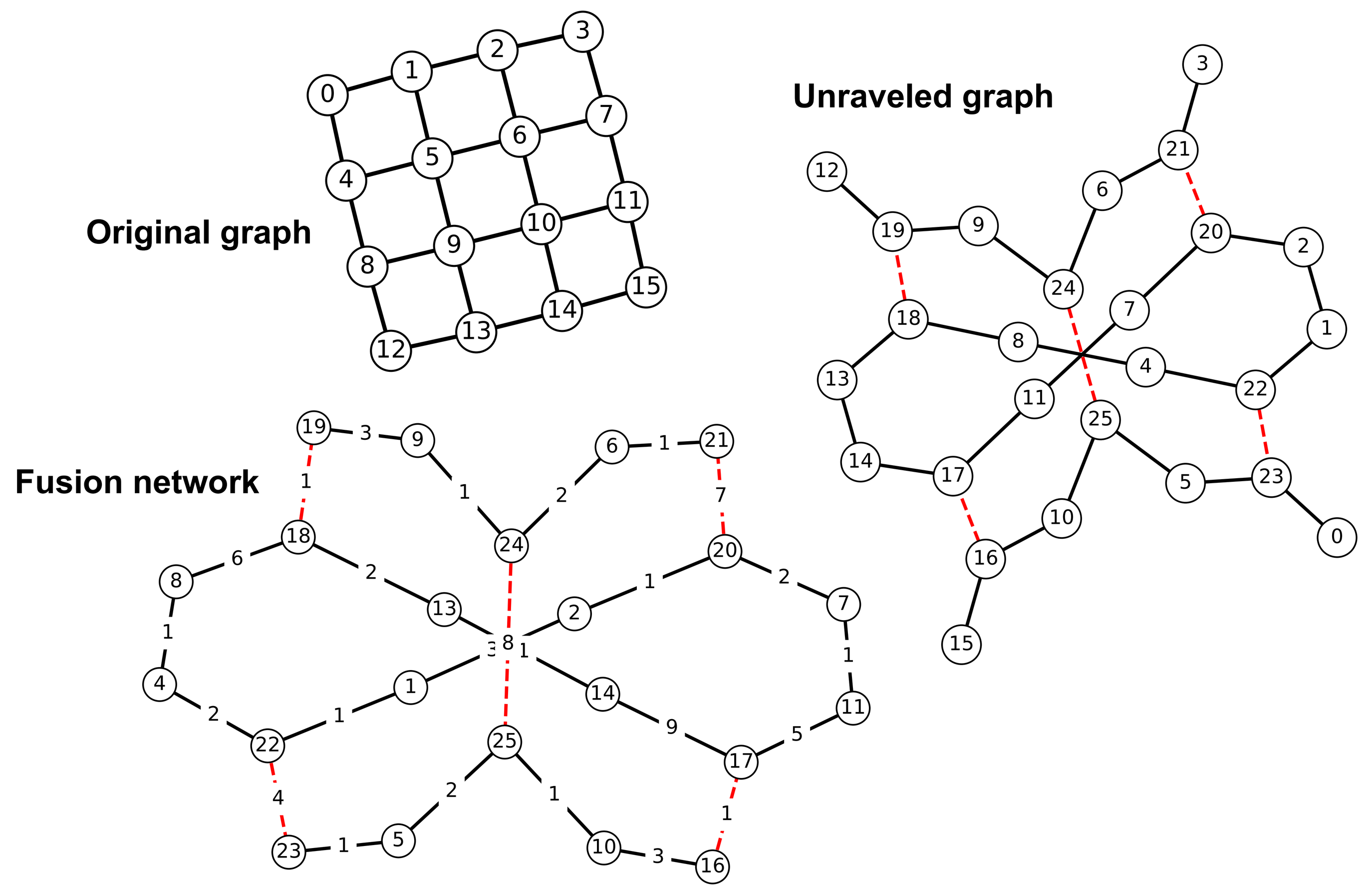
STAGE 2: CONSTRUCTION OF FUSION NETWORK



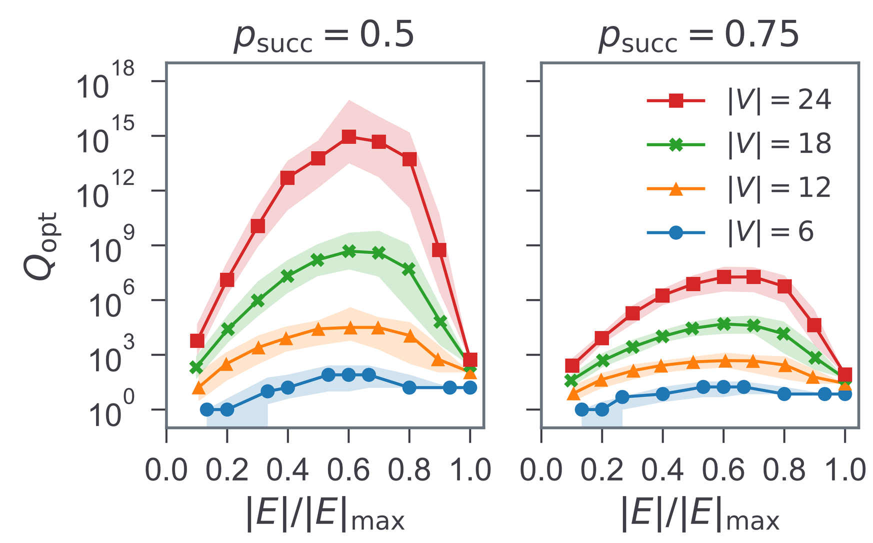
STAGE 3: DETERMINATION OF FUSION ORDER

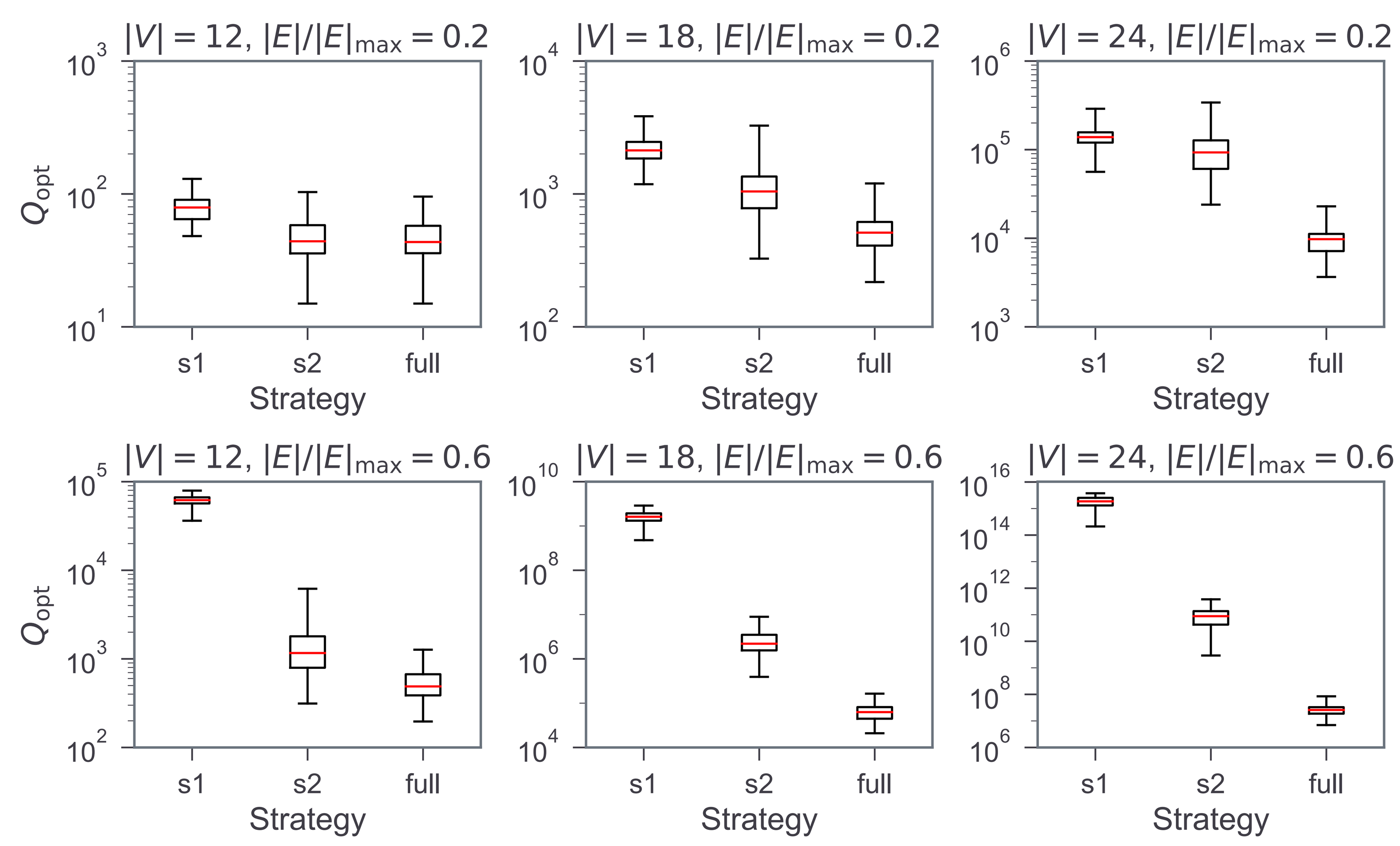


EXAMPLE



NUMERICAL RESULTS





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