UNIVERSAL RESOURCE-EFFICIENT TOPOLOGICAL MEASUREMENT-BASED QUANTUM COMPUTATION VIA COLOR-CODE-BASED CLUSTER STATES

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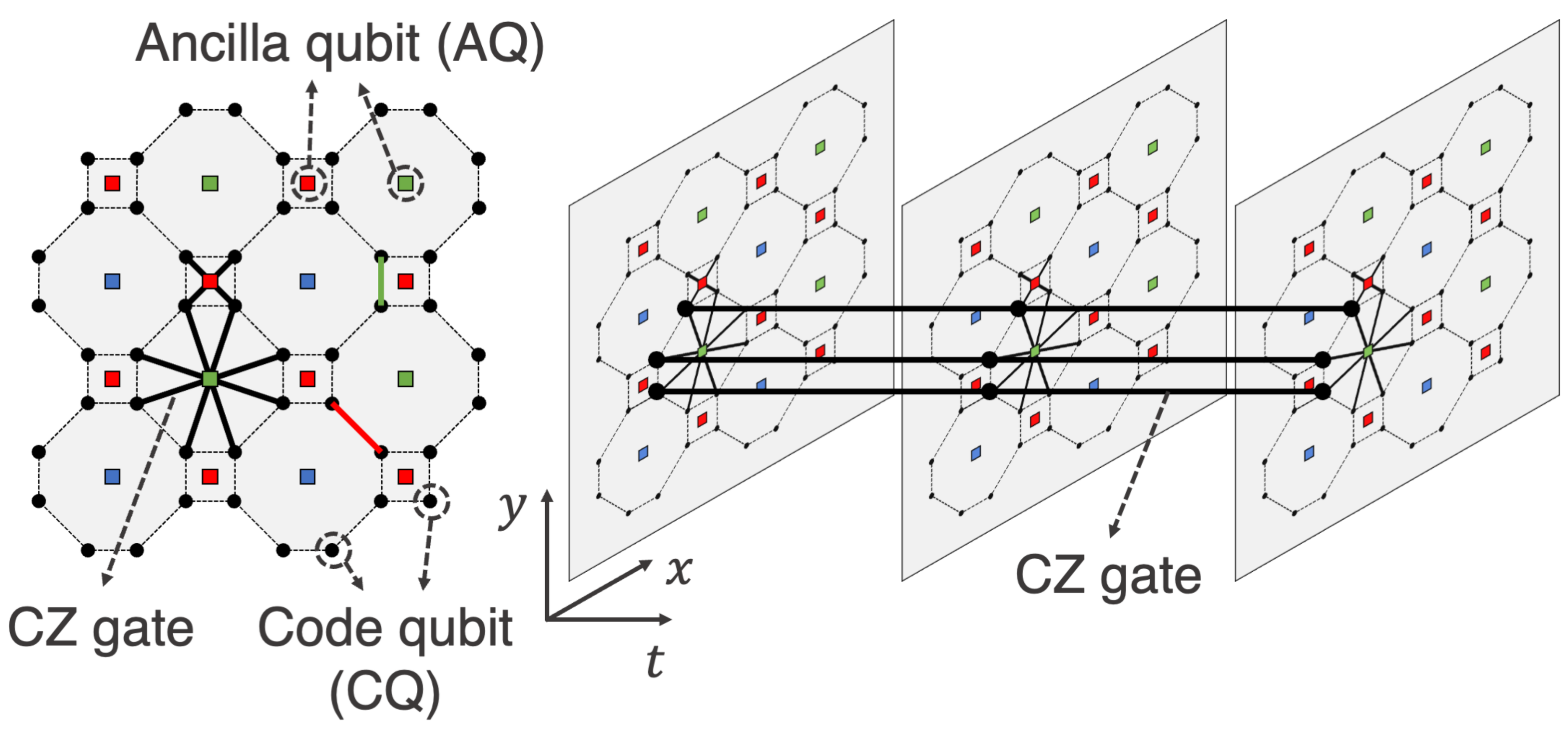
ABSTRACT

Topological measurement-based quantum computation (MBQC) enables universal fault-tolerant quantum computation (QC) via single-qubit Pauli measurements on cluster states [1–4]. Raussendorf's 3D cluster states (RTCSs) are mainly used for it, but then the logical Hadamard and phase gates require state distillation [3, 4] which generally consumes many ancillary qubits for fault-tolerant implementation [5]. We propose a new scheme utilizing another family of cluster states, called **color-code-based cluster states (CCCSs)**, with which **the Hadamard and phase gates can be performed fault-tolerantly without state distillation**. This leads to **fault-tolerant universal MBQC with significantly low resource overheads**, and it thus helps to remove a major obstacle against practical quantum computing.

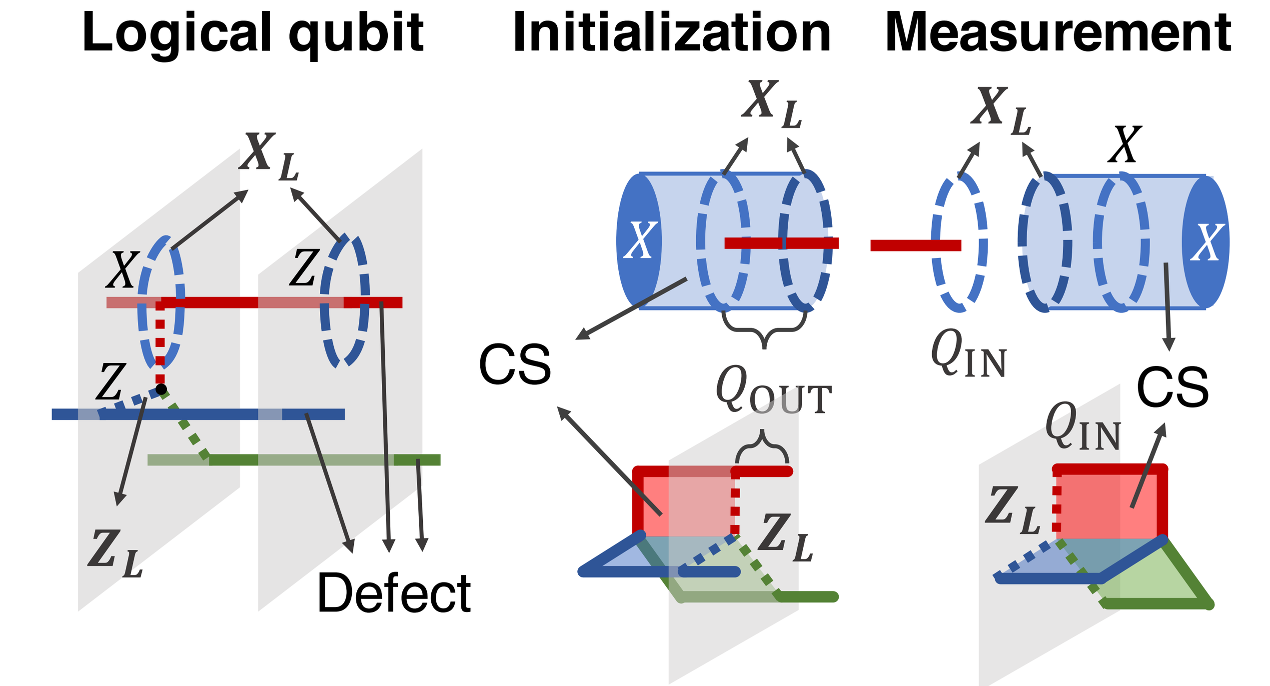
COLOR-CODE-BASED CLUSTER STATE

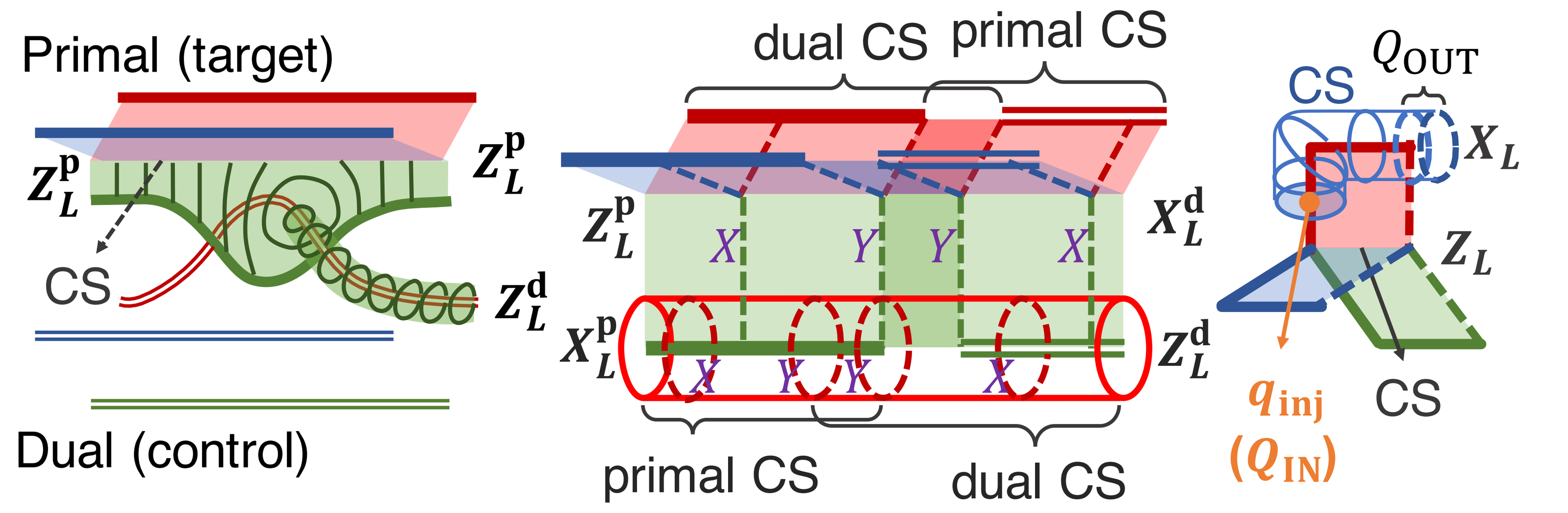


A **CCCS** is constructed by a foliation [6] of a 2D color-code lattice [7] $\mathscr{L}_{2\mathrm{D}}$. Multiple identical 2D layers are stacked along the simulating time (t) axis and each layer is derived from $\mathscr{L}_{2\mathrm{D}}$.

MBQC SCHEME

- **General process**: Input logical states are prepared in the input qubits. Other qubits are initialized to $|+\rangle := |0\rangle + |1\rangle$ states. Controlled-Z (CZ) gates are applied on given pairs of qubits. All the qubits except the output qubits are measured in a Pauli basis determined by the measurement pattern. The output logical state is obtained from the output qubits.
- **Measurement pattern**: Each qubit is either in the **vacuum** (X), in a **defect** (Z), in a **Y-plane** (Y), or an **injection qubits** (X).
- **Correlation surface (CS)**: A stabilizer composed of X operators in the interior of a surface and Z operators along its boundaries..
- Logical initialization, measurements, and gates are **implemented by deforming defects and placing Y-planes appropriately**. Y-planes are required for the Hadamard and phase gates.





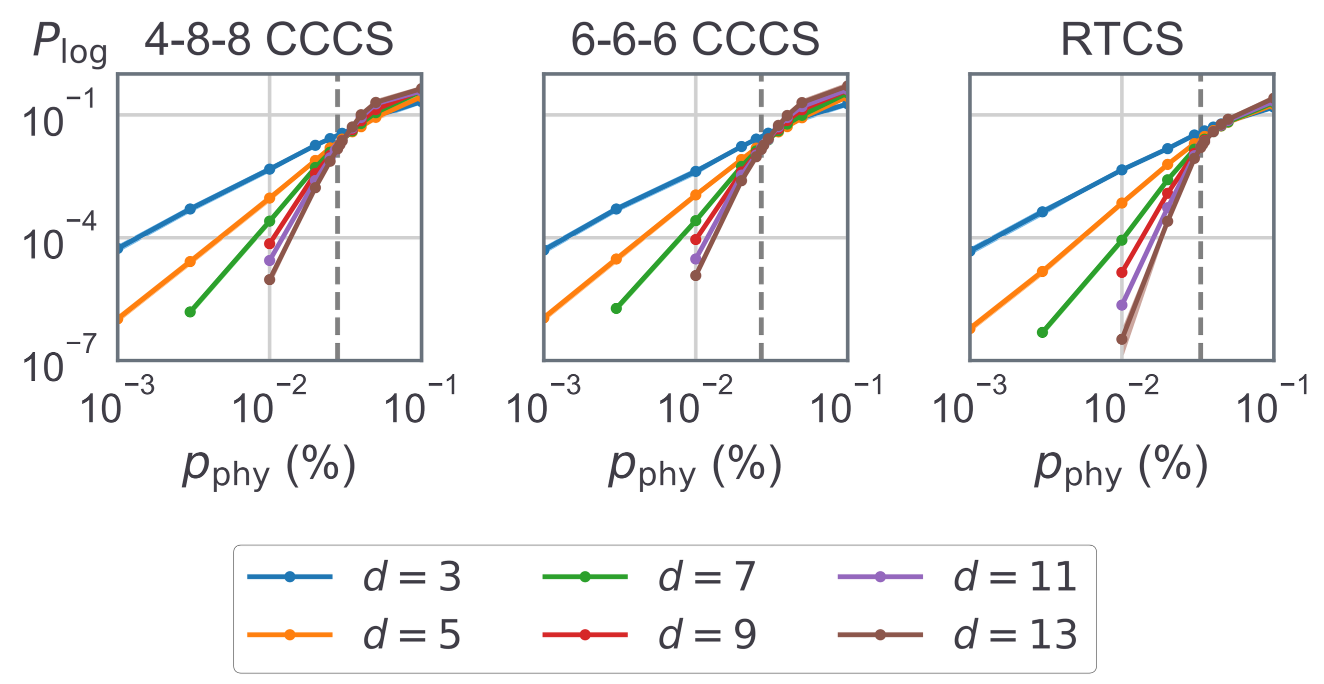
- **State injection**: Preparation of an arbitrary logical state from a single-qubit state. Required for the logical T gate.

ERROR CORRECTION

- Vacuum: **Parity-check operators** (PCs), which are stabilizers containing only X operators., are utilized.
- Defects: Some stabilizers having Z operators on defect qubits are additionally survived and utilized.
- Y-plane: **Hybrid PCs**, which are stabilizers containing X and Y operators, are utilized.
- Injection qubit: Inherently not fault-tolerant. Hence, the logical T gate requires state distillation to be fault-tolerant.

CALCULATIONS

Types of cluster states	n/k	N_{CZ}/k
RTCS	$\approx 6.6d^2$	$\approx 13.1d^2$
4-8-8 CCCS	$\approx 3.9d^2$	$\approx 10.5d^2$
6-6-6 CCCS	$\approx 3.7d^2$	$\approx 9.8d^2$



CONCLUSION

- We showed that **universal fault-tolerant QC is possible** with MBQC via CCCSs.
- Our scheme **consumes a significantly smaller amount of resources** than MBQC via RTCSs because of two reasons: 1. the logical Hadamard and phase gates do not require state distillation anymore and 2. the number of required physical qubits per logical qubit is ~1.8 times smaller.
- The error thresholds of both schemes are at a similar level.

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